Math 55 Discussion problems 28 Mar

1. Show that if m is a positive integer, then the probability that the m^{th} success occurs on the $(m+n)^{th}$ trial when independent Bernoulli trials, each with probability p of success, are run, is $\binom{n+m-1}{n} q^n p^m$.

2. Suppose that we roll a fair die until a 6 comes up.

   (a) What is the probability that we roll the die n times?

   (b) What is the expected number of times we roll the die?

3. Let X and Y be the random variables that count the number of heads and the number of tails that come up when two fair coins are flipped. Show that X and Y are not independent.

4. Show that if $X_1, X_2, ..., X_n$ are mutually independent random variables, then $E(\prod_{i=1}^{n} X_i) = \prod_{i=1}^{n} E(X_i)$.

5. Let $X(s)$ be a random variable, where $X(s)$ is a nonnegative integer for all $s \in S$, and let A_k be the event that $X(s) \geq k$. Show that $E(X) = \sum_{k=1}^{\infty} p(A_k)$.

6. Suppose that n balls are tossed into b bins so that each ball is equally likely to fall into any of the bins and that the tosses are independent.

   (a) Find the probability that a particular ball lands in a specified bin.

   (b) What is the expected number of balls that land in a particular bin?

   (c) What is the expected number of balls tossed until a particular bin contains a ball?

   (d) What is the expected number of balls tossed until all bins contain a ball? [Hint: Let X_i denote the number of tosses required to have a ball land in an i^{th} bin once $i-1$ bins contain a ball. Find $E(X_i)$ and use the linearity of expectations.]

7. What is the expected number of balls that fall into the first bin when m balls are distributed into n bins uniformly at random?